REPORT DOCUMENTATION PAGE	Form Approved OMB NO. 0704-0188

The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggesstions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA, 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any oenalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.
PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
15-02-2010	Final Report	15-Aug-2009 - 14-Apr-2010

4. TITLE AND SUBTITLE	5a. CONTRACT NUMBER
Structures with superelastic shape memory alloy stiffeners for maximuj damping and energy dissipation	W911NF-09-1-0423
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER 665803
6. AUTHORS Victor Birman	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES	8. PERFORMING ORGANIZATION REPORT NUMBER
Missouri University of Science and Technology Curators of the Univ. of Missouri on behalf of Missouri S&T 300 W. 12th Street Rolla, MO 65409 -	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211	10. SPONSOR/MONITOR'S ACRONYM(S) ARO 11. SPONSOR/MONITOR'S REPORT NUMBER(S) 56226-EG-II.1

12. DISTRIBUTION AVAILIBILITY STATEMENT

Approved for Public Release; Distribution Unlimited

13. SUPPLEMENTARY NOTES

The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT

The report presents an approach to passive vibration control of thin-walled structures utilizing shape memory alloy (SMA) “stringers.” The stringers represent prestressed superelastic SMA wires sliding within protective sleeves that are either embedded within the structure or bonded to its surface. The vibration control mechanism combines an effective elastic foundation reflecting the support provided by SMA wires to the structure with energy dissipation as a result of the hysteresis occurring in the wires. The second method of vibration control considered in

15. SUBJECT TERMS

smart structures, shape memory alloy, energy dissipation, damping

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Victor Birman
UU	UU	UU	UU		19b. TELEPHONE NUMBER 314-516-5436

Standard Form 298 (Rev 8/98)
Prescribed by ANSI Std. Z39.18

Report Title

Structures with superelastic shape memory alloy stiffeners for maximuj damping and energy dissipation

ABSTRACT

The report presents an approach to passive vibration control of thin-walled structures utilizing shape memory alloy (SMA) "stringers." The stringers represent prestressed superelastic SMA wires sliding within protective sleeves that are either embedded within the structure or bonded to its surface. The vibration control mechanism combines an effective elastic foundation reflecting the support provided by SMA wires to the structure with energy dissipation as a result of the hysteresis occurring in the wires. The second method of vibration control considered in the paper employs superelastic strings attached to the structure at discrete points and experiencing large-amplitude lateral vibrations as a result of near-resonant excitation conveyed from the structure. The potential advantage of this method is related to higher strain range in the strings resulting in more robust energy dissipation. As follows from numerical examples, the proposed methods offer significant passive damping and reductions in the vibration amplitudes.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

The paper based on the present report will be submitted to the Journal of Engineering Mathematics in May 2010 (invited issue).

Number of Papers published in peer-reviewed journals: 1.00

(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

Number of Papers published in non peer-reviewed journals: 0.00

(c) Presentations

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

(d) Manuscripts

Number of Manuscripts: 0.00

Number of Inventions:

Graduate Students

NAME	PERCENT_SUPPORTED
Ian Rusnak (part-time MS student)	0.00
FTE Equivalent:	**0.00**
Total Number:	**1**

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Victor Birman	1.00	No
FTE Equivalent:	**1.00**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME
Total Number:

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Summary of the Report

The report represents a draft of the journal paper that will be submitted to an archival journal in February-March 2010. The material will also be presented at one of the forthcoming conferences. For convenience, the outline of the effort is provided below:

1. The problem of dynamics of a plate (cross-ply symmetrically laminated plate or particular cases, such as an isotropic plate) with shape memory alloy (SMA) stringers was considered. The stringers represented SMA wires embedded within resin sleeves bonded to the plate and free to slide along the sleeves. The wires were statically pre-stretched (static offset). As a result, during vibrations of the plate, SMA wires experienced superelastic hysteresis with incomplete phase transformation.
2. The original idea of SMA stringers directly bonded to the plate in the manner similar to conventional stiffeners appeared unpractical since these stringers could not be statically prestressed. As a result, it was impossible to fully utilize the superelastic transformation and damping was insufficient.
3. The alternative design that was also considered in detail involved prestressed SMA wires supporting the plate within the span. This approach enabled us to utilize all advantages of the former approach (wires in sleeves), while resulting in a simple structural solution. Numerical results for the loss factor generated for this solution illustrated excellent damping of the plate-SMA wire system.

In conclusion, the study illustrated that “SMA stringers” can both enhance the stiffness acting similarly to a nonlinear elastic foundation or elastic supports and dramatically improve damping of the structure. While the solution for the enhanced stiffness was pioneered in the previous research of PI, the improvement of damping as well as the comprehensive theoretical formulation presented in this study represent a new development providing further justification to the use of SMA element in plate and shell structures.

Abstract

The paper presents an approach to passive vibration control of thin-walled structures utilizing shape memory alloy (SMA) “stringers.” The stringers represent prestressed superelastic SMA wires sliding within protective sleeves that are either embedded within the structure or

bonded to its surface. The vibration control mechanism combines an effective elastic foundation reflecting the support provided by SMA wires to the structure with energy dissipation as a result of the hysteresis occurring in the wires. The second method of vibration control considered in the paper employs superelastic strings attached to the structure at discrete points and experiencing large-amplitude lateral vibrations as a result of near-resonant excitation conveyed from the structure. The potential advantage of this method is related to higher strain range in the strings resulting in more robust energy dissipation. As follows from numerical examples, the proposed methods offer significant passive damping and reductions in the vibration amplitudes.

Keywords: Shape memory alloy, vibration control, hysteresis.

Analysis

1. Dynamics of Plates supported by Prestressed SMA Wires acting as an Elastic Foundation

Consider a plate subject to dynamic loading and supported by SMA wires in the sleeves that can be uniformly or nonuniformly distributed (Fig. 1). If a SMA wire is free to slide within the sleeve embedded within the structure or bonded to its surface it provides the support to the plate similar to that of an equivalent elastic foundation (Epps and Chandra, 1997). In this paper as well as in the subsequent papers by Birman (2007a, b) it was assumed that the range of strain being limited the wire remains in the austenitic phase and does not experience phase transformation. This assumption is not valid if the wire is prestressed to the strain corresponding to a partial transformation as is shown in Fig. 2. The motivation for such partial transformation is that it enables us to utilize the hysteresis corresponding to the range of strains imposed by the vibrating structure. Indeed, the typical strain range corresponding to the complete hysteresis is of an order of several percent (Seelecke et al., 2002; Zak et al. 2003), i.e. it requires the motion with exceedingly large amplitudes that seldom encountered in structural applications. Therefore, it is anticipated that the wire will experience a partial transformation during vibrations. Although the relevant strain range is limited and corresponds to an incomplete hysteresis loop, the energy dissipation may still be significant. Furthermore, if the wire is replaced with a string attached to the structure at one or several points, its vibration-induced strain range may significantly exceed that of the structure resulting in higher energy dissipation.

Introduction to the analysis: single SMA wire by the complex modulus approach

The stiffness of the foundation modeling the effect of a system of wires was derived by Epps and Chandra (1997) and expanded by Birman (2007a,b). Here we extrapolate the approach in the latter paper by accounting for the geometrically nonlinear effect. The bending moment acting at a cross section $x = \mu$ of a wire oriented in the x-direction, prestressed by a tensile force T_0 and subject to a concentrated force Q as shown in Fig. 3 is

$$M = \frac{Q(a-\mu)}{a}x - T\overline{w} - \left[Q(x-\mu)\right]_{x\geq\mu} \tag{1}$$

where $\overline{w}$ is a deflection of the wire from the straight position.

The tensile force in the wire includes both the static preload as well as the real-time dynamic contribution associated with the phase transformation. In this paper we employ the complex modulus approach to the characterization of SMA wire following Gandhi and Wolons (1999). According to this approach, the dynamic stress superimposed on the static prestress as a result of vibrations is given by (e.g., Nashif et al., 1985)

$$\sigma = E'\varepsilon \pm E''\sqrt{\varepsilon_0^2 - \varepsilon^2} \tag{2}$$

where the dynamic strain range is

$$\varepsilon = \varepsilon_0 \cos\omega t \qquad or \qquad \varepsilon = \varepsilon_0 \sin\omega t \tag{3}$$

In (3) ω is the driving frequency and t is time.

For a harmonic strain given by the first equation (3) the storage and loss moduli are evaluated as

$$\begin{aligned} E' &= \frac{\omega}{\pi\varepsilon_0}\int_0^{2\pi/\omega}\sigma(t)\cos\omega t \\ E'' &= \frac{\omega}{\pi\varepsilon_0}\int_0^{2\pi/\omega}\sigma(t)\sin\omega t \end{aligned} \tag{4}$$

Using eqns. (4) and experimental data, Gandhi and Wolons (1999) generated curves for the storage and loss moduli of the material as functions of the range (amplitude) of dynamic strain and the frequency of excitation.

If the wire vibration could be modeled as $w(x,t) = W(x)\cos\omega t$, the density of energy dissipation that occurs in the wire during one cycle of vibration is evaluated as (Nashif et al., 1985):

$$u_w = \pi E'' \varepsilon_0^2 \tag{5}$$

The amplitude of strain of the wire is related to the deflection of the structure that is equal to that of wire. If this deflection is known or prescribed, and represented as $w(x,t) = W(x)f(t)$ where the time function is normalized so that $\max|f(t)| = 1$,

$$\varepsilon_0 = \frac{1}{2a}\int_0^a (W,_x)^2\, dx \tag{6}$$

The tensile force during vibrations of the wire characterized by the first equation (3) can now be represented as

$$T = T_0 + (E'\cos\omega t \pm E''\sin\omega t)\varepsilon_0 A \tag{7}$$

where the sign depends on the sign of $\cos\omega t$ and A is the cross sectional area of the wire. In case where the excitation is given by the second equation (3), trigonometric functions in (7) are interchanged.

The negligible bending stiffness of the wire implies that the moment in (1) is equal to zero. Accordingly, (1) and (7) yield at $x = \mu$

$$Q = \frac{a_0}{(a-\mu)\mu}\left[T_0 + (E'\cos\omega t \pm E''\sin\omega t)\varepsilon_0 A\right]\overline{w}(\mu) \tag{8}$$

Representing the reaction of the wire by an elastic foundation one can write the stiffness at an arbitrary location x as

$$k(x,t) = \frac{Q}{\overline{w}(x)} = \frac{aT_0}{(a-x)x} + \frac{a}{(a-x)x}(E' \cos \omega t \pm E'' \sin \omega t)\varepsilon_0 A \tag{9}$$

The first term in the right side of (9) is identical to that derived by Birman (2007a) without accounting for dynamic deformations of the wire, while the second term represents the correction for these deformations. It is interesting to note that the stiffness of the foundation is time-dependent as a result of the dynamic stress generated in the wire.

It is obvious from (9) that the foundation provided by the SMA wire applies time-dependent harmonic reaction to the supported structure. In the previous studies (Epps and Chandra, 1997; Birman, 2007a) this dynamic component of the reaction was neglected. In the present paper our goal is to estimate a potential for energy dissipation. Accordingly, the time-dependent contribution to the foundation reaction is incorporated in the study.

The presence of the time-dependent term in the right side of (9) implies that the equation of motion of the structure affected by the product $k(x,t)w(x,t)$ will contain products $w(x,t)\cos \omega t$ and $w(x,t)\sin \omega t$. This means that the single-term harmonic excitation applied to the SMA wire results in a complicated time-dependent motion of the structure. In turn, this multi-harmonic motion forces the motion of the wire. Therefore, as long as the foundation affects the time-dependent response of the structure, this response cannot be approximated by a single-term harmonic motion, even if the driving force is a single-term harmonic function of time.

Note that in reality, the plate will be supported by SMA wire over a part of the span that does not include the boundaries, i.e. $a_1 \le x \le a_2$, Fig. 4. Besides technological difficulties designing the sleeves extended to the boundary points, such sleeves spanning over the entire length of the wire would produce prohibitively large local stresses at $x = 0$ and $x = a$ as is evidenced by the singularities of the terms in the right side of (9). Nevertheless, the points $x = a_1$ and $x = a_2$ being close to the edges we can assume that the mode shape of the SMA wire within $0 \le x \le a_1$ and $a_2 \le x \le a$ remains nearly identical to the deformed shape of the structure.

<u>Approach to the analysis of forced vibrations of a symmetrically laminated cross-ply plate supported by a system of parallel SMA wires</u>

In the following analysis we limit ourselves to the evaluation of the loss factor. However, if one aspires to analyze the response of the plate, the approach could be as follows. Let us assume that the pressure applied to the plate is given by

$$p(x, y, t) = p(x, y)\cos \omega t \tag{10}$$

The response of the plate to the applied pressure as well as to the reaction of the foundation (SMA wires) is sought in the form

$$w(x,y,t) = W(x, y)\sum_{n=1}^{N}\left(A_n \cos \omega nt + B_n \sin \omega nt\right) \tag{11}$$

Retaining a sufficient number of terms in series (11), the unknown amplitudes A_n and B_n should be determined from the equations of motion. The difficulty involved in employing such equations is related to the necessity to account for the energy dissipation in SMA wires. For example, if a cross-ply laminated thin plate is supported by two systems of SMA wires oriented along both couples of edges (Fig. 1), this equation becomes

$$\begin{aligned} &D_{11}w,_{xxxx} + 2(D_{12} + 2D_{66})w,_{xxyy} + D_{22}w,_{yyyy} + m(x, y)\ddot{w} + \left[C_1(x, y) + C_2(x, y)\right]\dot{w} \\ &+ \left[K_1(x, y, t) + K_2(x, y, t)\right]w = p(x, y, t) \end{aligned} \tag{12}$$

where bending stiffness coefficients D_{ij} are determined by customary equations, m is a mass of the plate per unit surface area, including the contribution of the wires, and $K_j(x, y, t)$ and $C_j(x, y)$ are the stiffness of the equivalent elastic foundation contributed by the SMA wires and their damping coefficient specified below, respectively. The contribution of the sleeves to the bending stiffness, stiffness of the equivalent elastic foundation and the damping coefficient could be incorporated through the Dirac delta function (e.g., Birman and Bert, 1990). It is anticipated that the contribution of resin sleeves to the bending stiffness will be negligible. While SMA wires that are free to slide in the sleeves do not contribute to the bending stiffness of the plate, their contribution to its mass should be accounted for. If the coordinates of the system of wires oriented in the x-direction and y-direction are denoted by y_k and x_n, respectively,

$$
\begin{aligned}
C_1 &= \sum_k C_{wx}\delta(y-y_k)\\
C_2 &= \sum_n C_{wy}\delta(x-x_n)\\
K_1 &= \sum_k k(x,t)\delta(y-y_k)\\
K_2 &= \sum_n k(y,t)\delta(x-x_n)\\
m(x,y) &= \rho_p h + \sum_n \left(\rho_w A_y + \rho_s A_{sy}\right)\delta(x-x_n) + \sum_k \left(\rho_w A_x + \rho_s A_{sx}\right)\delta(y-y_k)
\end{aligned}
\tag{13}
$$

where the stiffness terms are obtained from (9) for the wires oriented in the x-direction and from accordingly modified equations for the wires in the y-direction. Furthermore, ρ_p, ρ_w and ρ_s denote the mass density of the plate, wire and sleeve materials, respectively, and h is the thickness of the plate. The cross sectional areas of the wires oriented in the y and x directions and the cross sectional areas of the sleeves encompassing these wires are denoted by A_y, A_x, A_{sy} and A_{sx}.

The equivalent viscous damping of a system of wires could be determined from the requirement that the energy dissipation in the wire during a cycle of motion should be equal to the energy dissipation in the equivalent viscous damper. The former energy in a single wire can be evaluated from

$$
\Delta U_w = \left(\int_0^{2\pi/\omega} \sigma(\varepsilon)\frac{\partial \varepsilon}{\partial t}\,dt \right) V_w \tag{14}
$$

where the volume of wire is V_w. Alternatively, the area enclosed within the inner hysteresis loop can be obtained from experiments (e.g., Gandhi and Wolons, 1999).

The energy dissipated in an equivalent continuous viscous damper oriented in the x-direction along $y = y_k$ with the damping coefficient C_{wx} during one cycle of motion would be (Soedel, 1993)

$$
\Delta U'_w = \frac{1}{2}\int_0^{2\pi/\omega}\int_0^a C_{wx}\dot{w}^2(x,y_k,t)\,dx\,dt \tag{15}
$$

The requirement $\Delta U_w = \Delta U'_w$ results in the expression for the damping coefficient of the wire, i.e. C_{wx}.

If the wires oriented in the x- and y-directions are uniformly and closely spaced, their contribution to the equivalent elastic foundation stiffness, damping and mass per unit surface area are incorporated through $\frac{1}{l_y} \rightarrow \delta(x - x_n), \frac{1}{l_x} \rightarrow \delta(y - y_k)$ where $l_x = l_x(y)$ and $l_y = l_y(x)$ are the spacings of the systems of wires oriented in the y and x directions, respectively. Accordingly, the relevant terms in equation of motion (12) become

$$
\begin{aligned}
&K_1 = \frac{k_1}{l_x}, \qquad && K_2 = \frac{k_2}{l_x} \\
&C_1 = \frac{C_{wx}}{l_x}, \qquad && C_2 = \frac{C_{wy}}{l_y} \\
&m(x,y) = \rho_p h + \frac{\rho_w A_y + \rho_s A_{sy}}{l_y} + \frac{\rho_w A_x + \rho_s A_{sx}}{l_x}
\end{aligned}
\tag{17}
$$

The presence of SMA wires does not affect the boundary conditions. Accordingly, the analysis of a simply supported plate whose motion is characterized by the equation of equilibrium (12) or (16) can be conducted representing the function $W(x, y)$ in (11) in double Fourier series satisfying the boundary conditions.

<u>Relationship between the strain in the wire and the dissipated energy</u>

As was shown above, even if the driving load is a single harmonic function of time, the transverse motion of the plate and SMA wires should be modeled by a series of time functions. Therefore, it is necessary to accordingly modify that equation (5).

Consider the motion of the wire and plate that result in the strain in the SMA wire given by series

$$\varepsilon = \sum_{n=1}^{N} \left(\varepsilon_n^{(1)} \cos \omega nt + \varepsilon_n^{(2)} \sin \omega nt \right) \tag{18}$$

In the linear problem for each frequency number n the density of the dissipated energy is

$$u_w = \pi E''(\omega n)\varepsilon_0^2(\omega n) \tag{19}$$

where

$$\varepsilon_0^2(\omega n) = \left[\varepsilon_0^{(1)}(\omega n)\right]^2 + \left[\varepsilon_0^{(2)}(\omega n)\right]^2 \tag{20}$$

The total energy dissipated in the wire accounts for the dissipation of energy during one cycle of vibration. In the case of multi-harmonic motion, the period is defined by the lowest frequency, i.e. $T = \frac{2\pi}{\omega}$. The harmonic with the frequency ωn undergoes n cycles during this period. Accordingly, the total energy is

$$\Delta U_w = \pi V_w \sum_n n E''(\omega n)\varepsilon_0^2(\omega n) \tag{21a}$$

If the motion with the driving frequency ω is dominant, equation (21a) can be simplified to

$$\Delta U_w = \pi V_w E''(\omega)\varepsilon_0^2 \tag{21b}$$

The loss factor of the SMA wire is a ratio $\eta_w = E''/E'$. The loss factor of the system consisting of the plate and SMA wires can be evaluated using the approach suggested by Ungar and Kerwin (1962) as:

$$\eta = \frac{\eta_w U_w + \eta_p U_p + \eta_f U_f}{U_w + U_p + U_f} \tag{22}$$

where η_p is the loss factor of the plate and U_w and U_p are the maximum strain energy in the SMA wires in the longitudinal motion, and plate, respectively. The strain energy of the equivalent elastic foundation reflecting the reaction of the wire applied to the plate is denoted by U_f and the corresponding loss factor is η_f. Considering the dominant contribution of damping in the SMA wire it may be possible to neglect damping in the plate and in the equivalent elastic foundation provided by the wire ($\eta_p = \eta_f \approx 0$).

Consider the problem where the properties of the SMA wire, i.e. its loss and storage moduli are known. Let us assume that the strain amplitude in the wire, i.e. ε_0, is prescribed. Then the energy dissipated in the wire is determined by (21). The maximum strain energy in a single SMA wire is $U'_w = {\Delta U_w}/{\eta_w}$. It remains to specify the maximum strain energies of the plate and of the equivalent elastic foundation provided by the wire that corresponds to this strain amplitude. The following analysis concentrates on the case of a symmetrically laminated cross-ply simple supported large aspect ratio plate that bends into a cylindrical surface during forced vibrations. The plate is supported by a system of equally-spaced wires oriented along short edges (in the x-direction).

The maximum strain energy of the equivalent elastic foundation provided by closely-spaced SMA wires is specified below. Let the motion of the plate be represented by the same relation as that leading to equation (6), i.e.

$$w = W(x) f(t) \tag{23}$$

where $\max|f(t)| = 1$.

The mode shape of a large aspect ratio simply supported vibrating plate is represented by

$$W(x) = \sum_{s=1.} W_s \sin \frac{\pi s x}{a} \tag{24}$$

The relationship between the maximum per-cycle strain in the wire and the corresponding deflections of the plate is specified substituting deflection mode shape (24) into (6). Then the integration yields

$$\varepsilon_0 = \frac{\pi^2}{4} \sum_s s^2 \left(\frac{W_s}{a}\right)^2 = \frac{\pi^2}{4} \left(\frac{W_1}{a}\right)^2 \sum_s s^2 n_s{}^2 \tag{25}$$

where $n_s = {W_s}/{W_1}$.

The maximum value of the foundation strain energy corresponds to

$$U_f = \frac{1}{2l_x a}\int_0^a k(x,t)W^2(x)dx \tag{26}$$

The foundation provided by SMA wires in (26) includes only the time-dependent contribution, excluding the constant tensile force from the support. Being interested only in the maximum value of the tensile strain energy in the SMA wire we can express the maximum dynamic tensile stress from the condition

$$U'_w = \frac{1}{2}\sigma_0\varepsilon_0 V_{wire} \tag{27}$$

Accordingly, for the known maximum strain ε_0, the maximum stress in the wire is

$$\sigma_0 = \frac{2U'_w}{\varepsilon_0 V_{wire}} \tag{28}$$

This implies that the maximum strain energy of the unit-width foundation provided by a system of SMA wires is (see (9), (21b), (25), (26) and (28)):

$$U_f = \frac{\pi E''\varepsilon_0 A}{l_x \eta_w} W_1^2 \sum_s \sum_r n_s n_r \left[F_{sr}(a_2) - F_{sr}(a_1)\right] \tag{29}$$

The term in (29) that depends on cosine and sine integral is given by (Mathematica 2009):

$$F_{sr}(x) = \frac{1}{2a}\left[\begin{array}{l} Ci\left(\frac{\pi(r-s)x}{a}\right) - Ci\left(\frac{\pi(r+s)x}{a}\right) - \cos(\pi(r-s))Ci\left(\frac{\pi(r-s)(a-x)}{a}\right) + \\ \cos(\pi(r+s))Ci\left(\frac{\pi(r+s)(a-x)}{a}\right) - \sin(\pi(r-s))Si\left(\frac{\pi(r-s)(a-x)}{a}\right) + \\ \sin(\pi(r+s))Si\left(\frac{\pi(r+s)(a-x)}{a}\right) \end{array}\right] \tag{30}$$

where $a_1 \le x \le a_2$ is the section of the plate supported by the wire.

The substitution of W_1^2 from (25) into (29) yields

$$U_f = \frac{4E''Aa^2\varepsilon_0^2}{\pi l_x \eta_w \sum_l l^2 n_l^2} \sum_s \sum_r n_s n_r \left[F_{sr}(a_2) - F_{sr}(a_1)\right] \tag{31}$$

The maximum strain energy of the unit-width symmetrically laminated cross-ply plate bending into a cylindrical surface is

$$U_p = \frac{1}{2}\int_0^a \left[D_{11}(w,_{xx})^2\right]dx \tag{32}$$

which upon the substitution of (24) yields

$$U_p = \frac{\pi^4}{4a} D_{11}\left(\frac{W_1}{a}\right)^2 \sum_s s^2 n_s^2 \tag{33}$$

In terms of the strain amplitude in the wire, the plate energy is

$$U_p = \frac{\pi^2}{a} D_{11}\varepsilon_0 \tag{34}$$

The maximum strain energy of a system of SMA wires per unit width of the plate is obtained by (21b) as

$$U_w = \frac{\pi E''\varepsilon_0^2 V_w}{l_x \eta_w} \tag{35}$$

The analysis of the loss factor in the plates supported by a system of parallel SMA wires can be conducted as follows. Using a feasible mode shape of motion, we can estimate the corresponding ratios $n_s = \frac{W_s}{W_1}$. Then for a prescribed value of the maximum strain ε_0 the maximum strain energy per unit plate width are obtained from (31), (34) and (35). Subsequently, the loss factor of the structure (plate and SMA wires) is given by (22). Note that even if the mode shape of motion is known, the loss factor is affected by the magnitude of the maximum strain (or by the maximum deflection). This reflects a nonlinear nature of the problem. As discussed above, the only loss factor accounted for is that contributed by SMA wires. This is a conservative

assumption but it can always be modified if the loss factors of the plate and the equivalent elastic foundation are known.

Alternative support system: point-wise plate support by superelastic SMA wires

According to this approach, the structure is supported by stretched superelastic SMA wires at selected points, rather than continuously (Fig. 5). The method was considered by Birman (2007a,b) and found quite effective. However, the effect of the hysteresis in SMA wires on damping was not analyzed in this paper. In this paragraph, we illustrate the application of the method to the case where a cross-ply symmetrically laminated large aspect ratio plate experiences dynamic bending forming a cylindrical shape. The plate is simply supported and a number of closely-spaced SMA wires are connected to the plate at the midspan (see Fig. 5). The goal is to estimate the loss factor of such structure accounting for hysteresis of superelastic SMA wires.

Contrary to the previous solution, in the present case it appears possible to derive a closed-form expression for the loss factor of the structure. Given the maximum deflection of the mid-span of the plate, W, the amplitude of the dynamic reaction force per plate unit-width applied by SMA wires to the plate is

$$R = 4\sigma_0 \frac{A}{al_x} W \tag{36}$$

The corresponding maximum energy of the support per unit width of the plate provided by wire is

$$U_f = 2\sigma_0 \frac{A}{al_x} W^2 \tag{37}$$

Substituting (28) and (21b) into (37) and using the relationship

$$\varepsilon_0 = \frac{1}{2}\left(\frac{2W}{a}\right)^2 \tag{38}$$

we obtain

$$U_f = \frac{2\pi E'' V_w}{l_x \eta_w} \varepsilon_0^2 \tag{39}$$

The energy dissipated in SMA wires per unit plate width is

$$U_w = \frac{\pi E'' \varepsilon_0^2 V_w}{\eta_w l_x} \tag{40}$$

Note that $U_f = 2U_w$.

The strain energy of the plate vibrating according to (24) is given by

$$U_p = \frac{1}{16}\left(\frac{\pi}{a}\right)^4 D_{11} a^3 \varepsilon_0 \sum_s s^2 n_s^2 \tag{41}$$

Substituting (39), (40) and (41) into (22) we obtain the closed-form solution for the loss factor.

$$\eta = \frac{\pi E''(\omega) V_w \varepsilon_0}{3\pi E'(\omega) V_w \varepsilon_0 + \frac{1}{16}\left(\frac{\pi}{a}\right)^4 D_{11} a^3 l_x \sum_s s^2 n_s^2} \tag{42}$$

Note that the loss factor is a nonlinear function of the amplitude of the strain.

Numerical results

The effectiveness of SMA wires supporting the midspan of a large aspect ratio plate shown in Fig. 5 was illustrated using experimental data of Gandhi and Wolons (1999). The loss factor of the structure consisting of the plate and a system of parallel SMA wires was generated as a function of the ratio of the maximum-per-cycle strain energy in the plate to that in the system of wires (this ratio is shown along the horizontal axes in Figs. 6 and 7). The loss factor is shown in Fig. 6 for the case where the static strain offset in SMA wires was equal to 3.93% for three strain amplitudes equal to 1%, 2% and 5% and the frequency of motion equal to 6Hz. As follows from Fig. 6, the larger fraction of the strain energy in SMA wires results in a higher loss factor. The loss factor increases with a larger amplitude of strains but this advantage becomes

less pronounced and even reverses once the strain amplitude reaches 2%. Such result is acceptable since high strain amplitude of the plate is unlikely in applications.

It is instructive to compare the loss factor shown in Fig. 6 to the loss factor of representative materials shown in Table 1. As follows from this comparison, even at the strain amplitude of 1%, SMA wires can provide excellent damping as long as the structure is designed so that the maximum strain energy in the plate does not exceed that in SMA wires by a factor of 5 or more.

Table 1. Loss factors of conventional materials

Materials	Loss factor
Aluminum	2×10^{-5} to 2×10^{-3}
Concrete	0.02 to 0.06
Glass	0.001 to 0.002
Rubber	0.1 to 1.0
Steel	0.002 to 0.01
Wood	0.005 to 0.01

Source: Pan and Cho, 2007.

The results shown in Fig. 7 illustrate the effect of static offset (static strain) on the loss factor of the plate with SMA wires (Fig. 5) vibrating with the dynamic strain amplitude equal to 2.28% at a relatively low frequency of 0.2Hz. As follows from this figure, the loss factor becomes smaller at larger static offset. However, as was noted by Gandhi and Wolons (1999), a very small static pre-strain is undesirable since it may result in the SMA wire vibrating outside the superelastic hysteresis strain range and a lower damping. Therefore, it is necessary to maintain a minimum static offset to ensure that the wire experiencing a superimposed dynamic strain vibrates within the total strain range corresponding to the hysteresis loop.

Conclusions

SMA wires embedded in sleeves and continuously supporting the plate as well as SMA wires supporting the plate at discrete points have been analyzed. As follows from numerical examples generated for the latter case, such wires can drastically enhance damping of the structure. The increase in damping is particularly significant if the static prestress of the wire is kept to a necessary minimum. Furthermore, larger amplitude of dynamic strains increases damping. The increase in the ratio of the maximum-per-cycle strain energy of the plate to that of the SMA wires is counterproductive. In general, a designer should avoid the situation where this ratio exceeds a factor of 5.

References

Birman, V., 2007a. Shape Memory Elastic Foundation and Supports for Passive Vibration Control of Composite Plates, International Journal of Solids and Structures, Vol.45, pp. 320-335.

Birman, V., 2007b. "Enhancement of Stability of Composite Plates using Shape Memory Continuous and Point Supports," AIAA Journal, Vol. 45, pp. 2584-2588.

Birman, V. and Bert, C.W., 1990. "Dynamic Stability of Reinforced Composite Cylindrical Shells in Thermal Fields," Journal of Sound and Vibration, Vol.142, pp. 183-190.

Epps, J., Chandra R., 1997. "Shape Memory Alloy Actuation for Active Tuning of Composite Beams," Smart Materials and Structures 6, 251-264.

Gandi, F., Wolons, D., 1999. "Characterization of the Pseudoelastic Damping Behavior of Shape Memory Alloy Wires using Complex Modulus," Smart Materials and Structures 8, 49-56.

Nashif, A.D., Jones, D.I.G. and Henderson, J.P., 1985. Vibration Damping, Wiley, New York.

Mathematica, 2009. Online Integrator. http://integrals.wolfram.com.

Muller, I. and Xu, H., 1991. "On the Pseudo-Elastic Hysteresis," Acta Metallurgical Mat., Vol. 39, pp. 263-271.

Pan, Q. and Cho, C., 2007. "The Investigation of a Shape Memory Alloy Micro-Damper for MEMS Applications," Sensors, Vol. 7, pp. 1887-1900.

Savi, M.A. and Paiva, A., 2005. “Describing Internal Subloops due to Incomplete Phase Transformation in Shape Memory Alloys,” Archives of Applied Mechanics, Vol. 74, pp. 637-647.

Seelecke, S., Heintze, O., Masuda, A., 2002. Simulation of Earthquake-Induced Structural Vibrations in Systems with SMA Damping Elements,” In: Proceedings of the SPIE Smart Structures & Materials Conference. Vol. 4697, 238-245.

Soedel, W., 1993. Vibrations of Shells and Plates. Marcel Dekker, New York.

Ungar, E.E. and Kerwin, E.M, Jr., 1962. “Loss Factors of Viscoelastic Systems in Terms of Strain Energy,” Journal of the Acoustical Society of America, Vol. 34, pp. 954-958.

Zak, A.J., Cartmell, M.P., Ostachowicz, W.M., Wiercigroch, M., 2003. “One-dimensional Shape Memory Alloy Models for Use with Reinforced Composite Structures,” Smart Materials and Structures 12, 338-346.

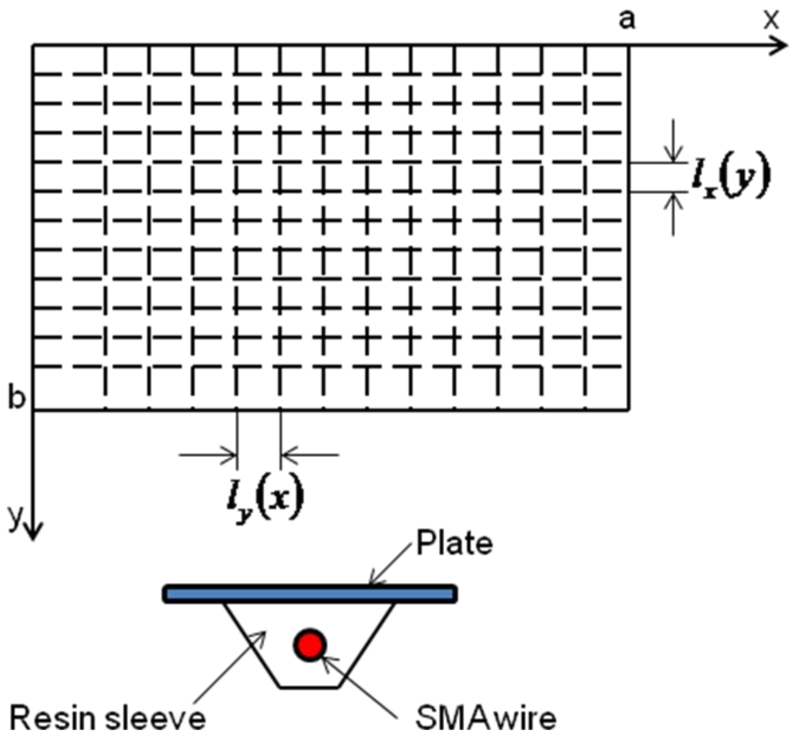

Fig. 1. Plate supported by two mutually perpendicular systems of parallel SMA wires in sleeves.

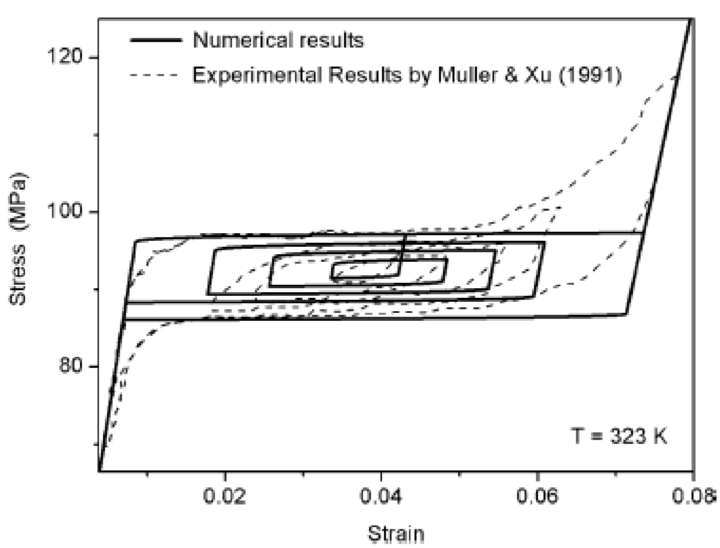

Fig. 2. Numerical and experimental pseudoelastic subloops corresponding to incomplete phase transformation. Numerical results are generated by Savi and Paiva (2005), experimental results are from Muller and Xu (1991). From Savi and Paiva (2005).

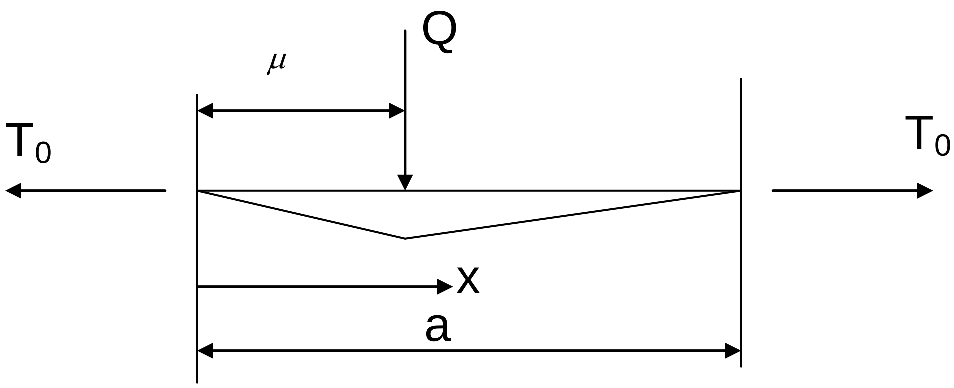

Fig. 3. SMA wire subject to a concentrated force Q.

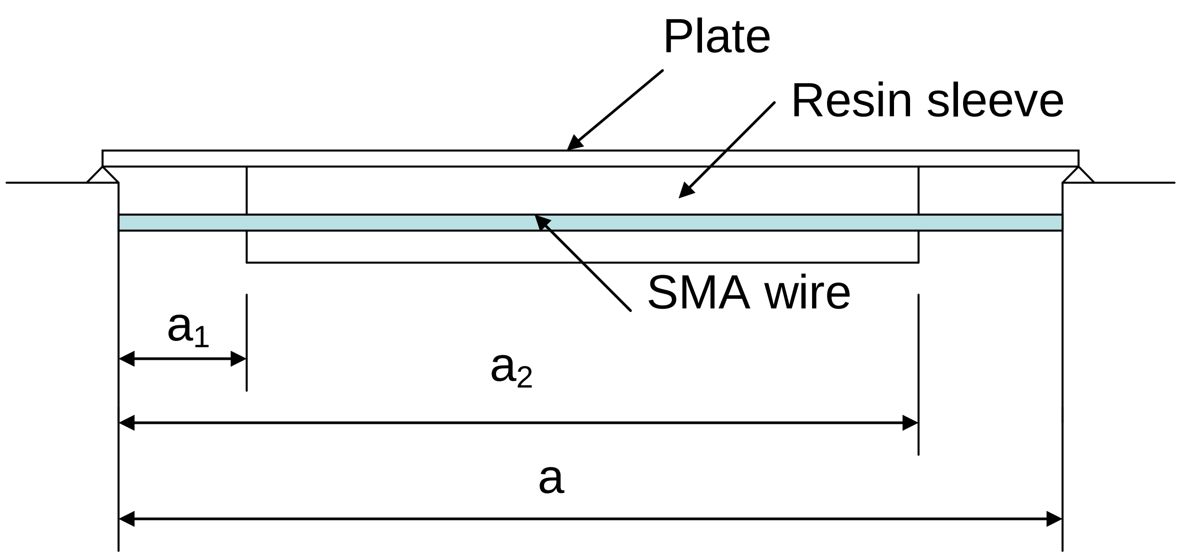

Fig. 4. SMA wire supporting plate through the sleeve in the region $a_1 < x < a_2$.

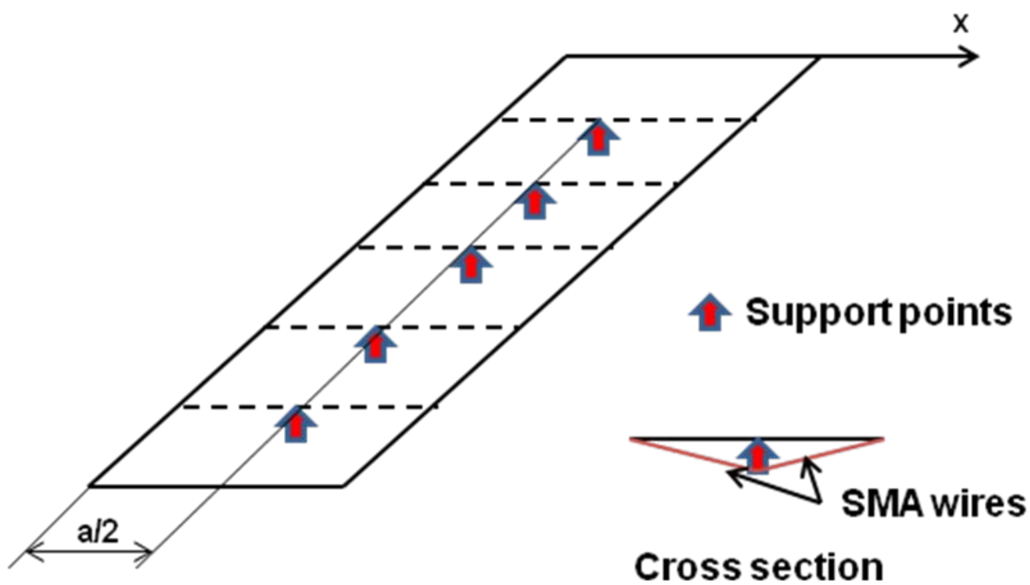

Fig. 5. SMA wires oriented along the x-axis and supporting the large aspect ratio plate at the midspan.

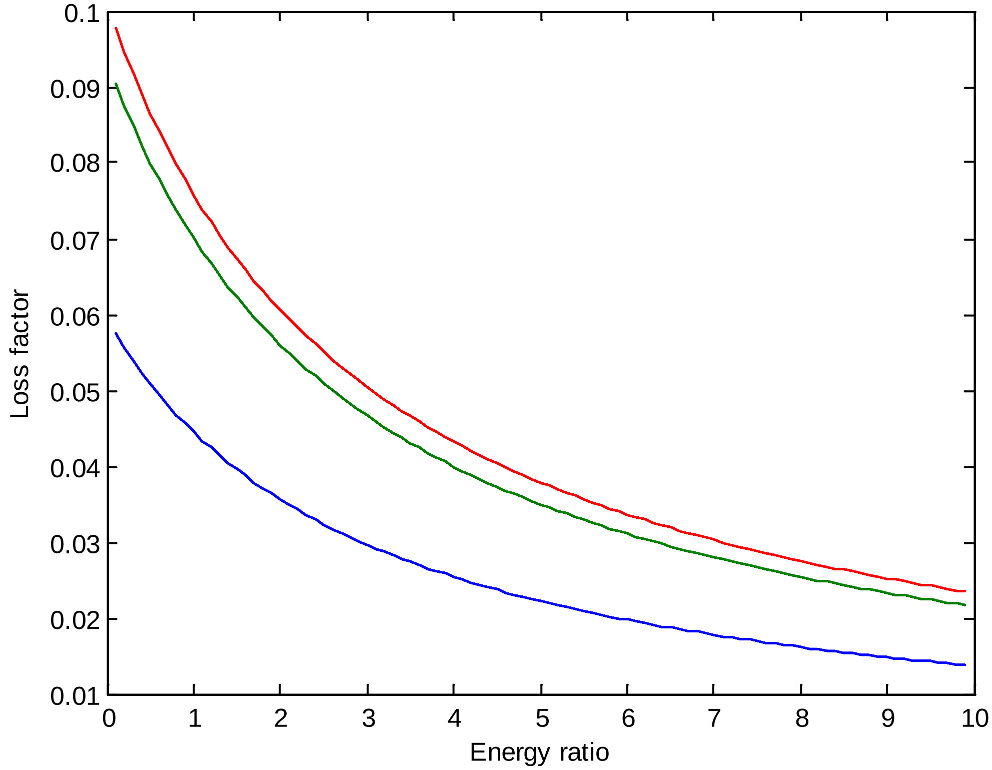

Fig. 6. Loss factor of a large aspect ratio plate supported by SMA wires at the midspan. Blue: $\varepsilon_0 = 1\%$, red: $\varepsilon_0 = 2\%$, green $\varepsilon_0 = 5\%$.

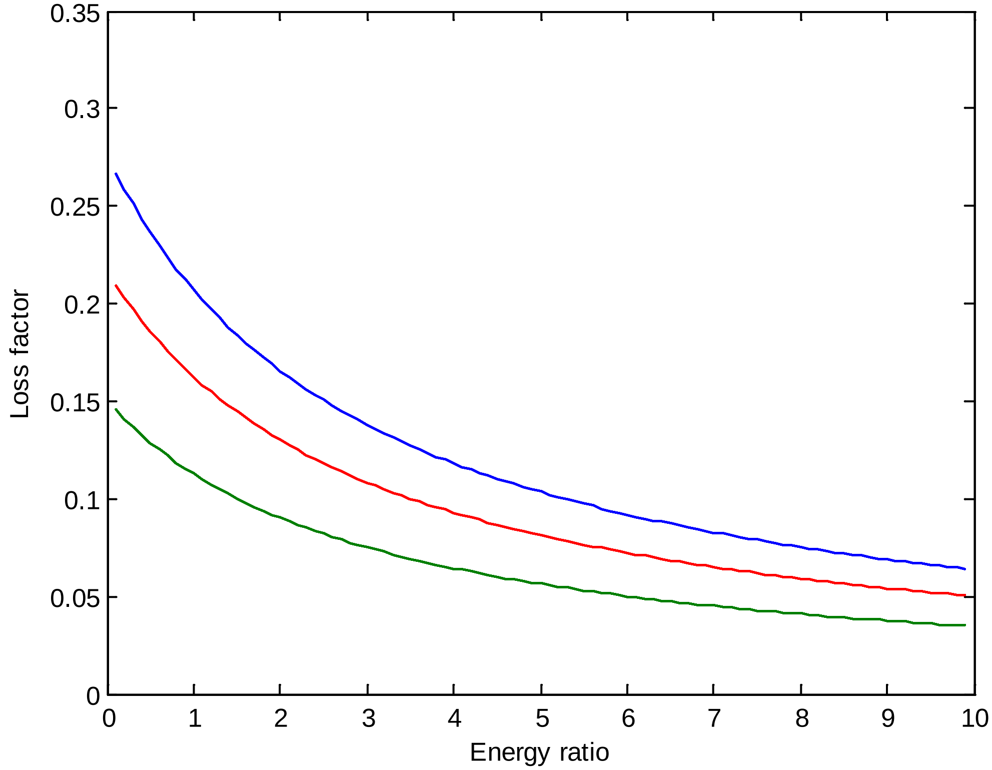

Figure 7. Loss factor of a large aspect ratio plate supported by SMA wires at the midspan as a function of the static prestress. Blue: 2.5%, red: 3.5%, green: 4.5%.